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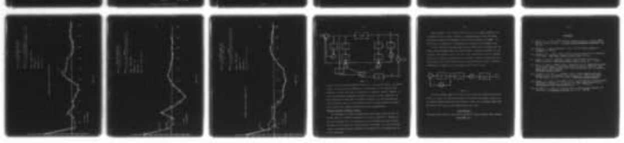

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10 Kumpati S. Narendra ~~and~~ Lena S. Valavani

S ~~/~~ IS Report No. 7711

11 April, 1978

12 32p.

Contract N00014-76-C-0017

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14 S/IS-7711

Systems + Inform Science

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DIRECT AND INDIRECT ADAPTIVE CONTROL

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ABSTRACT

The paper considers the control of an unknown linear time-invariant plant using Direct and Indirect Control. Using a specific controller structure and the concept of positive realness, adaptive laws which are identical are derived for the two cases. The stability questions that arise are also shown to be the same. Simulation results are presented towards the end of the paper to demonstrate the effectiveness of the new scheme proposed.

Keywords. Adaptive control; direct control; indirect control; stability.

INTRODUCTION: The control of an unknown linear time-invariant plant has remained an open problem for a long time and in recent years there have been many attempts to resolve it (Landau, 1972; Åström and Wittenmark, 1973; Monopoli, 1974; Feuer and Morse, 1977; Narendra and Valavani, 1977). The methods used for the resolution of the adaptive control problem can be broadly classified as (i) Indirect Control and (ii) Direct Control methods. In the former, the parameters and/or state variables of the unknown plant are estimated and these, in turn, are used to adjust the parameters of a controller. Such control systems have also been referred to as self tuning regulators (Åström and Wittenmark, 1973) in the literature. In direct control there is no explicit identification of the plant but the control parameters are adjusted so that the error between plant output and that of a reference model (called the desired output) tends to zero asymptotically. Direct control systems have also been called model reference adaptive systems (Landau, 1972).

No matter which method is used, the difficulties associated with the control problem arise from two sources. The first stems from the fact that the plant is essentially a black box and only the input is accessible for control purposes. This raises the question, which is strictly algebraic in nature, as to whether a controller structure exists which can generate the appropriate control input $u(t)$. It is now well-known that the answer to this question, whether direct or indirect control is used, is positive.

The second problem, which is purely analytic in nature, is concerned with the generation of adaptive laws and proving that the control parameters, starting from arbitrary initial conditions, will converge to desired values, consequently resulting in the output error signal tending to zero asymptotically. While most of the adaptive controllers described in the literature are locally asymptotically stable, very little was done until recently in the design of such globally stable controllers.

Monopoli (1974) suggested an ingenious method of adaptive control using an augmented error signal. However, the basic question of stability was left unanswered

in that paper. The possibility of the adaptive loop becoming unstable therefore existed and became the subject of discussion of subsequent papers by Feuer and Morse (1977) and Narendra and Valavani (1977). Feuer and Morse (1977) suggested a complex controller structure and showed the stability of the adaptive loop. Unfortunately, the motivation for the choice of the controller structure as well as the development of the stability proofs are rather difficult to follow. As the authors readily admit, their results are primarily to establish the existence of a solution rather than to have a practical method for the adaptive control of an unknown plant.

Narendra and Valavani (1977) proposed a general approach for the adjustment of the control parameters using the concept of positive realness which resulted in a simple controller structure. The stability problem was clarified in that paper and a conjecture was made regarding the uniform asymptotic stability of the overall adaptive loop. In this paper, a unified approach to direct and indirect control is presented. It is shown, that by the proper choice of the observer and controller structures the two approaches can be made identical. The stability problem for indirect control is consequently the same as that for the case of direct control and the conjecture made earlier applies here as well.

The new scheme is considerably simpler to implement than that suggested previously by the authors. The simulation results included in the paper indicate that the method is sufficiently robust to be used in real applications. In addition to enjoying practical advantages, the method proposed here also reveals the close connection that exists between two philosophically different approaches; the direct control of an unknown plant using a reference model and sensitivity functions requires the same information needed to identify the plant using indirect control.

INDIRECT AND DIRECT CONTROL PROBLEMS: The plant that is to be controlled has the input-output pair $\{u(t), y_p(t)\}$ which can be modeled by a linear time-invariant system described by the differential equations:

$$\dot{x}_p = A_p x_p + b_p u(t)$$
$$y_p = h^T x_p \tag{1}$$

Where A_p is an (nxn) matrix, h_p and b_p are constant n-vectors and the transfer function $W_p(s)$ of the plant may be represented as:

$$W_p(s) \triangleq h^T(sI-A_p)^{-1}b_p = k_p \frac{Z_p(s)}{R_p(s)} \tag{2}$$

$W_p(s)$ is strictly proper with $Z_p(s)$ a monic Hurwitz polynomial of degree m, $R_p(s)$ a monic polynomial of degree n and k_p a constant gain parameter. We also assume that only m,n and the sign of k_p are known for the design of the controller.

The output behavior expected from the plant, when it is suitably augmented by a controller, is described by $y_M(t)$, the output of a model M. The model has a reference input r(t) which is piecewise continuous and uniformly bounded and a transfer function denoted by $W_M(s)$ where

$$W_M(s) \triangleq \frac{k_M Z_M(s)}{R_M(s)} \tag{3}$$

$Z_M(s)$ in (3) is a monic Hurwitz polynomial of degree m, $R_M(s)$ is a monic Hurwitz polynomial of degree n and k_M is a constant. The error between plant output and the desired model output is defined by

$$e_1(t) \triangleq y_p(t) - y_M(t) \tag{4}$$

The principal requirement on the controller that has to be designed is that it be free of differentiators. This, in turn, implies that the model transfer function $W_M(s)$ must be such that

$$\text{(number of poles)} - \text{(number of zeros)} \geq n - m$$

[While the model transfer function can contain n_1 poles and m_1 zeros where $n_1 - m_1 \geq n - m$, we shall for simplicity assume throughout this paper that $n_1 = n$ and $m_1 = m$].

Indirect Control: The basic philosophy of this approach is to estimate the parameters of the unknown plant from input-output data and, in turn, use these estimates to adjust the parameters of a controller such that the transfer function of the controlled plant evolves to that of a model.

Direct Control: Unlike the previous case, no explicit plant identification is involved in direct control. However, explicit use of a model described by (3) is used and the generated error $e_1(t)$ is directly used in the adaptive control laws.

The adaptive control problem using indirect or direct control may now be stated as follows: Given a plant P with an input-output pair $\{u(t),y_p(t)\}$ determine a suitable control input u(t) such that

$$\lim_{t\to\infty} |e_1(t)| = \lim_{t\to\infty} |y_p(t) - y_M(t)| = 0 \tag{5}$$

where $y_M(t)$ is a desired output.

The principal difference between indirect and direct control as stated in this paper lies in the following two facts:

(i) a model of the desired behavior is explicitly used in direct control while a model of the plant identified on line is used in indirect control.

(ii) identification error in indirect control and control error in direct control are used to update the control parameters.

In the following sections we shall discuss the close relation that exists between the controller structure, adaptive laws and stability questions that arise in these two approaches.

A Simple Case of Direct Control: The general method of direct control discussed by Narendra and Valavani (1977) is best explained in terms of the simple configuration shown in Fig. 1 with a single adjustable parameter k(t).

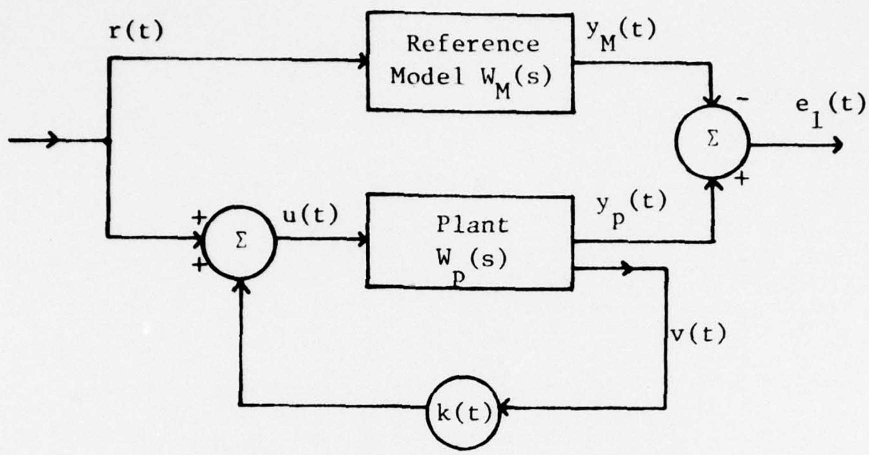

Fig. 1

The reference model has a positive real transfer function $W_M(s)$. The plant has a transfer function $W_p(s)$ and the controller contains a single adjustable parameter $k(t)$ as shown in Fig. 1 so that the input to the plant $u(t) = r(t) + k(t)v(t)$. It is also known that for some constant value k^* of k, the transfer function of the plant with the controller matches that of the model exactly.

If $k(t) = k^* + \phi(t)$, the system governing the error $e_1(t)$ may be represented as shown in Fig. 2. The problem now is to adjust $\phi(t)$ such that $e_1(t)$ tends to zero asymptotically with time. It is well-known that with such a configuration, the

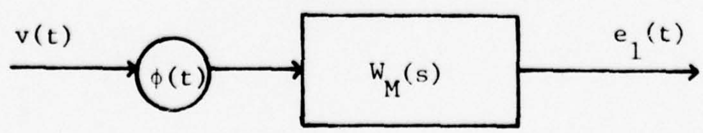

Fig. 2

adaptive law

$$\dot{k}(t) = \dot{\phi}(t) = -e_1(t)v(t) \tag{6}$$

yields bounded signals $e_1(t)$ and $\phi(t)$ for any input $v(t)$ (whether or not it is bounded). If $v(t)$ is bounded, $\lim_{t\to\infty} e_1(t) = 0$ or the error $e_1(t)$ tends to zero. Further, if $v(t)$ is "sufficiently rich" in addition, Morgan and Narendra (1977) have shown that $\phi(t) \to 0$ as $t \to \infty$, or the parameter error also tends to zero.[1]

In the problem under consideration, if the output of the model is bounded, the boundedness of $e_1(t)$ implies the boundedness of $y_p(t)$ and hence $v(t)$. Hence $e_1(t) \to 0$ as $t \to \infty$. Further, if the reference input is sufficiently rich, $k(t) \to k^*$.

In the above analysis the principal simplification arises from the fact that the reference model is positive real. The more realistic case arises when the model is stable but is not positive real. In such a case the parameter k is replaced by $P_L(k)$ as shown below. [For a more detailed presentation of this operator the reader is referred to Narendra and Valavani (1977)].

If $L(s)$ is a Hurwitz polynomial of degree n_1 in the differential operator 's' and $\theta(t)$ is any bounded differentiable function of time, the operator $P_L(\theta)$ is defined as

$$P_L(\theta) = L(s)\theta(t)L^{-1}(s) \ . \tag{7}$$

From the definition of $P_L(\theta)$ it follows that it is (i) linear in $\theta(t)$, (ii) $P_L(\theta^*) = \theta^*$ for a constant θ^*, (iii) $P_L(\theta) - \theta = P_L(\phi) - \phi$, where $\theta(t) = \theta^* + \phi(t)$.

If $L(s)$ is such that $W(s)L(s)$ is positive real, the parameter $k(t)$ in the feedback loop can be replaced by $P_L(k(t))$ and the same arguments can be used as before to determine the adaptive laws. In this case

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1 It is a commonly observed fact that in most adaptive control situations the parameter errors do not tend to zero. Qualitatively this may be accounted for by the fact that as the adaptation proceeds the input $u(t)$ to the plant ceases to be "sufficiently rich".

$$\dot{k}(t) = e_1(t)\zeta(t) \tag{8}$$

where $\zeta(t) = L^{-1}(s)v(t)$.

The use of the operator L(s) in the above analysis implies the use of n_1 derivatives in the feedback loop. Since the controller is to be free of differentiators, the following procedure which involves feeding an auxiliary input to the model (free of differentiators) was suggested.

The adaptive laws given in (8) can be generated when k(t) in Fig. 1 is replaced by $P_L(k(t))$. In Fig. 3 the signal k(t)v(t) is fed back into the plant and the signal $[k(t) - P_L(k(t))]v(t)$ is fed back into the model resulting in an error equation which is identical to that obtained in the earlier case and hence the adaptive laws have the same form given in (8).

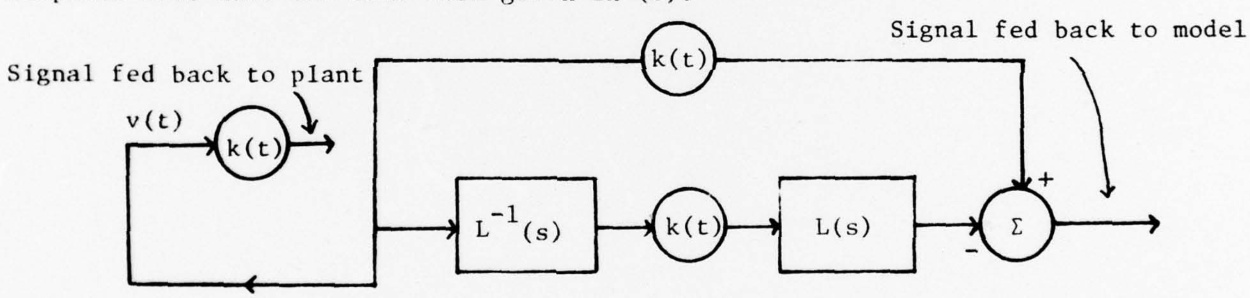

Fig. 3

It may appear at first sight that we have merely shifted the use of differentiators to the model from the plant since $[k(t) - P_L(k(t))]$ involves differentiations. However, since the model (which is transparent) has n poles and m zeros and the degree of L(s) is $n_1 = n - m - 1$, the operator $L(s)W_M(s)$ can be realized without the use of differentiators by the proper choice of the model, as shown in Fig. 4.

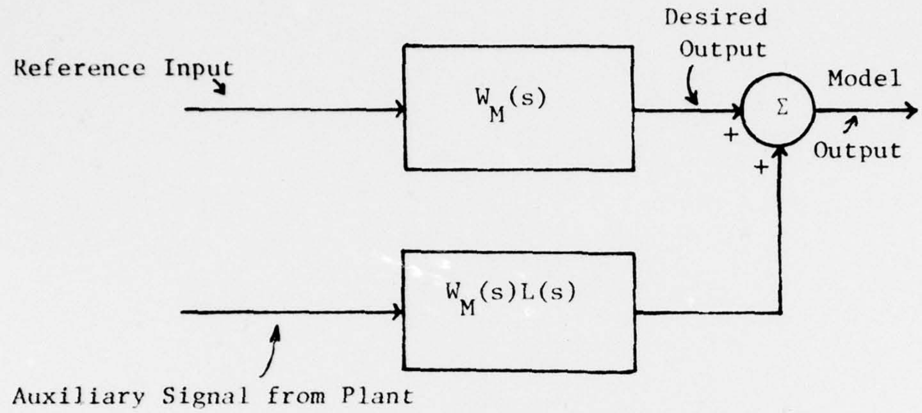

Fig. 4

It is this procedure with some modifications that was suggested by Narendra and Valavani (1977) for the adaptive control of an unknown plant. In the following sections the extension of the above ideas to the general direct control problem and its relation to the indirect control problem are examined.

The Observer Structure for Indirect Control:

As mentioned earlier, the observer plays a central role in the indirect control problem. The parameters of the plant are continuously estimated and these estimates, in turn, are used to determine the control parameters of the system. The rationale behind such an adjustment is that, when the identification parameters tend to their true values, the control parameters will also approach their desired values for which the transfer function of the feedback loop will match that of a specified reference model. Narendra and Valavani [1976] showed that the above approach generally leads to nonlinear stability problems which are intractable. The principal difficulty in such cases arises when attempting to relate estimates of the identification parameters to those of the control parameters.

In this section, we suggest an observer structure which yields a simple relation between observer and controller parameters. In indirect control a reference model is not explicitly used to generate the desired output but is contained implicitly in the rule for updating the control parameters. As shown in this section, the simplification that results from using the new observer structure is partly due to the specific manner in which the reference model is embedded in the adaptive observer.

The following lemma and corollaries are needed to justify the observer structure.

Lemma 1: Given the relatively prime polynomials p and q of degrees m ($\leq n-1$) and n respectively, with q monic, a monic polynomial Γ and a polynomial Δ of degree $(n-1)$ exist such that the polynomial

$$\Gamma q + \Delta p = M$$

where M is any $(2n-1)$ degree monic polynomial. (This lemma was also used in the Direct Control Problem in [6]).

Proof: Since p and q are relatively prime, polynomials α and β of degrees $(m-1)$ and $(n-1)$, where $m \geq 1$, exist such that

$$\alpha q + \beta p = 1.$$

(For a detailed proof of the above, the reader is referred to [6], Lemma 2). Let M be the monic polynomial of degree ($\leq$) $2n-1$. Then

$$M\alpha q + M\beta p = M$$

If N is the proper part of $\frac{\beta M}{q}$,

$$\frac{\beta M}{q} = N + \frac{\Delta}{q}, \text{ where } \Delta \text{ is of degree } (\leq)\ n-1.$$

We then have

$$(M\alpha + Np)q + (M\beta - Nq)p = M$$

where $M\alpha + Np$ is a monic polynomial of degree n-1 and $M\beta - Nq = \Delta$ is of degree $\leq (n-1)$. Choosing $M\alpha + Np = \Gamma$, the result follows.

Corollary 1: If M_1 is any monic polynomial of degree $(2n+m-1)$ such that $M_1 = M\sigma$ where σ is a monic polynomial of degree m, polynomials Γ_1 and Δ_1 of degrees $(m+n-1)$ exist which satisfy

$$\Gamma_1 q + \Delta_1 q = M_1$$

This follows directly from Lemma 1 by choosing $\Gamma_1 = \Gamma\sigma$ and $\Delta_1 = \Delta\sigma$.

Corollary 2: Given two polynomials $\rho(s)$ and $q_M(s)$ of degrees $(n-m-1)$ and n respectively, if p(s) and q(s) are relatively prime polynomials of degrees m and n, then the rational function p(s)/q(s) can be expressed as

$$\frac{p(s)}{q(s)} = \frac{\Gamma(s)}{q_M(s)\rho(s) + \Delta(s)} \tag{9}$$

by the proper choice of the polynomials $\Gamma(s)$ and $\Delta(s)$ of degree $(n-1)$.

Proof: Equation (9) implies $\Gamma q - \Delta p = p(s)q_M(s)\rho(s)$ and since the right hand side is of degree $(2n-1)$ the lemma applies directly.

Representation of Plant Transfer Function:

From Corollary 2 it follows that if a plant has a transfer function p(s)/q(s) with m zeros and n poles, it can be represented in an equivalent form as shown in Figure 5b, with $\Lambda(s) = p_M(s)\rho(s)$ and $\rho(s)$ an $(n-m-1)$ degree polynomial, $\Gamma^*(s), \Delta^*(s)$ and $\Lambda(s)$, $(n-1)$ degree polynomials in 's' and a fixed part in the forward loop with transfer function $p_M(s)/q_M(s)$ where $p_M(s)$ and $q_M(s)$ are polynomials in 's' of the same degree as p(s) and q(s). The basic structure of the adaptive observer as derived from Figure 5b is shown in 5c. The polynomials $\Gamma(s)$ and $\Delta(s)$ are adjusted

using input and output data so that $\Gamma \to \Gamma^*$ and $\Delta \to \Delta^*$ asymptotically.

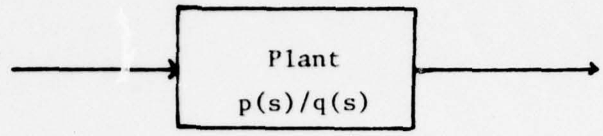

Figure 5a

The Plant

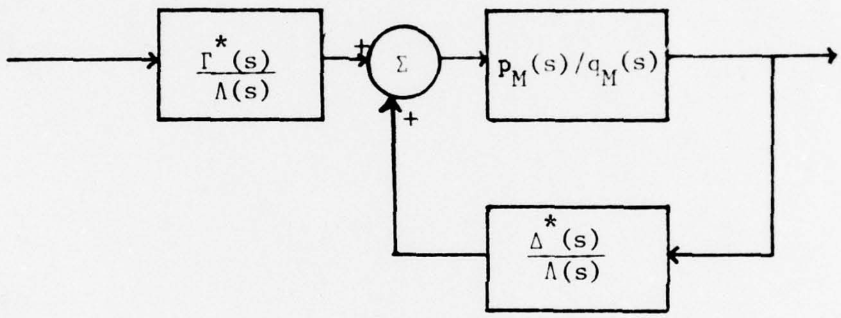

Figure 5b

An Equivalent Representation of the Plant

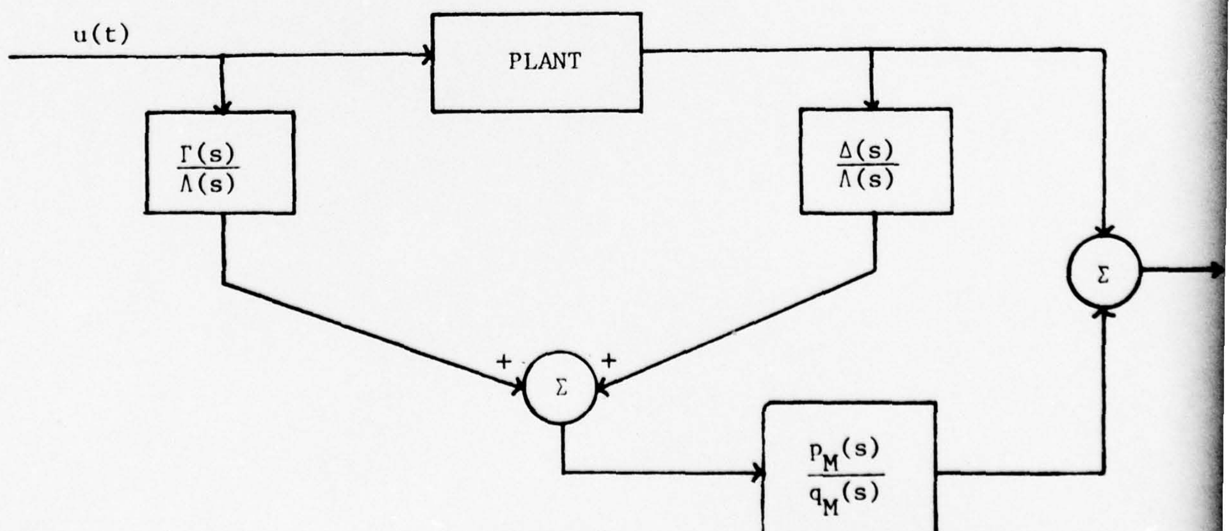

Figure 5c

Structure of Adaptive Observer

The rational function $\frac{p_M(s)}{q_M(s)} = W_M(s)$ used in the observer represents the transfer function of the reference model. The aim of the control problem is to generate a suitable control signal u(t) to the plant so that the plant output $y_p(t)$ evolves asymptotically towards the desired output $[W_M(s)]r(t)$. We notice that instead of using an explicit reference model we embed it as a fixed part of the observer.

In the following sections it is shown that the observer structure shown in Figure 5c is adequate for the control of a plant which has (n-1) zeros and n poles and the reference model has a positive real transfer function. In such a case the indirect and direct control problems are found to be identical. For more general cases this structure is modified as described later in this paper.

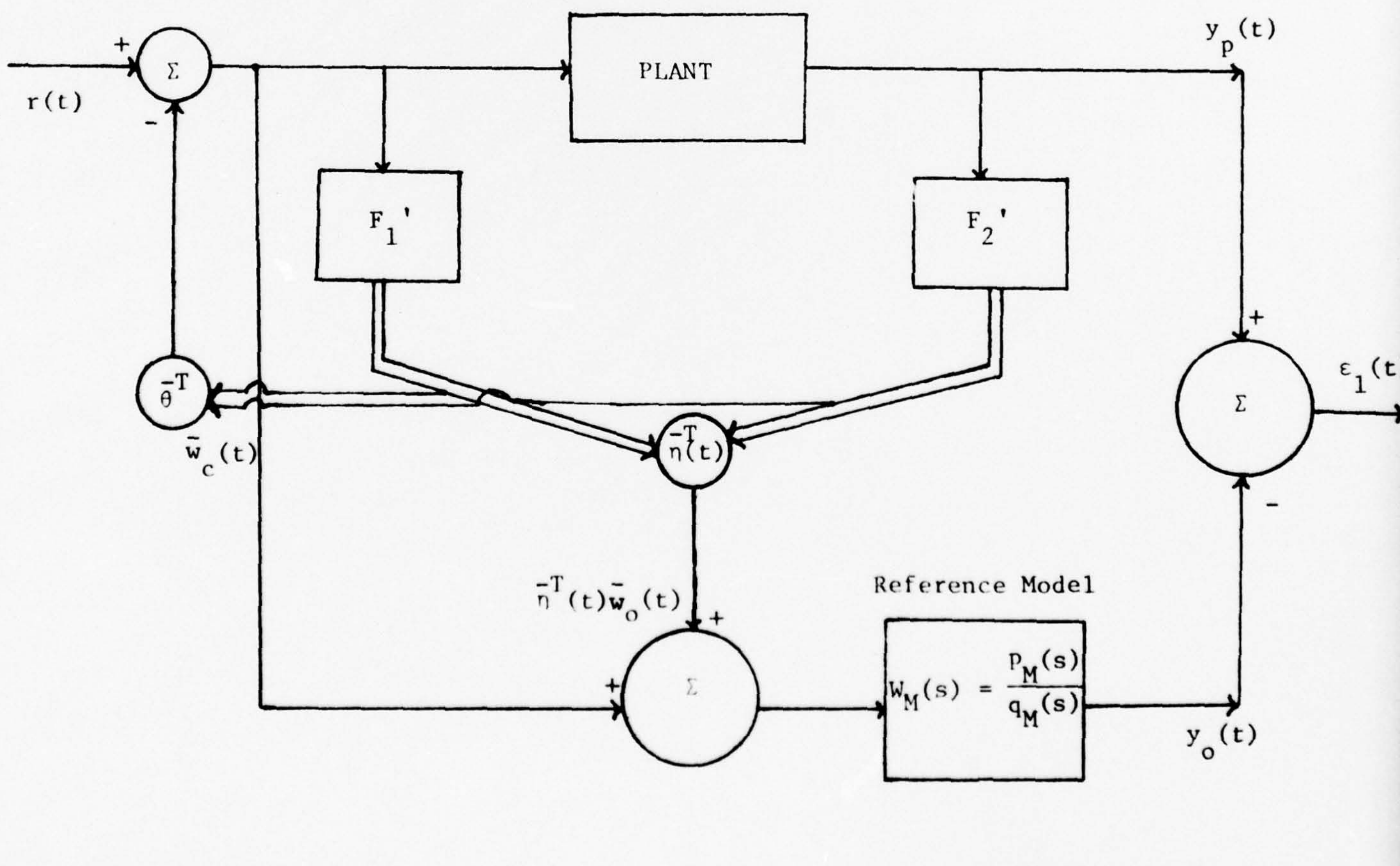

Figure 6

The observer consists of two auxiliary signal generators F_1' and F_2' described by the (n-1)th order vector differential equations

$$\dot{v}^{(1)} = \Lambda v^{(1)} + bu \qquad\qquad (10)$$
$$w_o^{(1)} = \gamma_0 u + \gamma^T v^{(1)} \qquad (F_1')$$

$$\dot{v}^{(2)} = \Lambda v^{(2)} + bu \qquad\qquad (11)$$
$$w_o^{(2)} = \delta_0 y_p + \delta^T v^{(2)} \qquad (F_2')$$

and

$$b^T = [0,0....,1]$$

where Λ is an (n-1)x(n-1) stable matrix. F_1' and F_2' have transfer functions $R_1(s)$ and $R_2(s)$ respectively for constant values, denoted by (*), of the parameters γ_i and δ_i $(i = 0,1,...,n-1)$.

$$R_1(s) = \gamma_0^* + \gamma^{*T}(sI-\Lambda)^{-1}b \triangleq \frac{\Gamma^*(s)}{\Lambda(s)} \qquad (12)$$

$$R_2(s) = \delta_0^* + \delta^{*T}(sI-\Lambda)^{-1}b = \frac{\Delta^*(s)}{\Lambda(s)} \qquad (13)$$

If Λ is in companion form, γ_i^* and δ_i^* $(i = 1,2,...,n-1)$ represent the coefficients of s^{n-i-1} of the numerator polynomials $\Gamma^*(s)$ and $\Delta^*(s)$ respectively, and γ_0^* and δ_0^* the gains associated with the signals $u(t)$ and $y_p(t)$. The 2n signals $u(t), v^{(1)}(t), y_p(t)$ and $v^{(2)}(t)$ are used in the identification procedure and the corresponding 2n parameters used in the observer are the elements of a parameter vector $\eta(t)$. Defining

$$w_o^T(t) \triangleq [u(t), v^{(1)^T}(t), y_p(t), v^{(2)^T}(t)] \qquad (14)$$

$$\eta^T(t) \triangleq [\gamma_0(t), \gamma^T(t), \delta_0(t), \delta^T(t)]$$

the estimate $\hat{y}_p(t)$ of the output of the plant $y_p(t)$ is realized in the observer as the output of the reference model M when subjected to an input $\eta^T(t)w_o(t)$ or

$$y_p(t) = [W_M(s)][\eta^T(t)w_o(t)]$$ [1]

The identification error $\epsilon_1(t)$ is then defined as the error between plant and observer outputs:

$$\epsilon_1(t) \triangleq y_p(t) - \hat{y}_p(t).$$

The Controller Structure: The controller (for both direct and indirect control) uses (2n-1) of the signals generated by the observer along with the reference input r(t) to generate the input u(t) to the plant. If the vector $w_c(t)$ is defined as

$$w_c^T(t) \triangleq [r(t), v^{(1)^T}(t), y_p(t), v^{(2)^T}(t)] \tag{15}$$

and the control parameter vector by $\theta(t)$ where

$$\theta^T(t) \triangleq [c_0(t), c^T(t), d_0(t), d^T(t)], \tag{16}$$

the input u(t) to the plant is denoted by

$$u(t) = \theta^T(t)w_c(t).$$

From equations (14) and (15) it is seen that the (2n-1) signals $v^{(1)}(t), y_p(t)$ and $v^{(2)}(t)$ are common to both observer and controller. In addition to these, as indicated in equation (15), the controller uses the reference input r(t) to generate u(t), while the observer uses the plant input u(t) to generate $\hat{y}_p(t)$.

Comment 1: For direct control, an explicit reference model is used and the control parameter vector $\theta(t)$ is adjusted on-line using the control error $e_1(t)$. In indirect control the identification parameter vector $\eta(t)$ is adjusted on-line using the identification error $\epsilon_1(t)$ and these, in turn, are used to adjust $\theta(t)$. Since the control error $e_1(t)$ is not available in this case, the entire adaptation is carried

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[1] Throughout the paper both differential equations and transfer functions are used in the arguments and, depending on the context, 's' is used as a differential operator or the Laplace transform variable.

out using only $\varepsilon_1(t)$.

Direct and Indirect Control:

We shall assume in the following sections, for simplicity, that the gain k_p of the plant transfer function is known. In such a case the gain $c_0 = \frac{k_M}{k_p}$ is a constant and only (2n-1) parameters in the controller have to be adjusted. Further, since $\gamma_0 = k_p$ is a constant in the observer, only (2n-1) parameters have to be adjusted to identify the system. For convenience, we shall assume that $k_M = k_p = 1$ so that $c_0 = \gamma_0 = 1$ and denote by $\bar{\theta}(t), \bar{\eta}(t), \bar{w}_o(t) = \bar{w}_c(t)$ the (2n-1) dimensional controller and observer parameter and input vectors used in generating the adaptive laws.

Reference Model with Positive Real Transfer Function:

The importance of the specific observer structure, discussed in the previous section, containing the reference model as an integral part, becomes evident when we consider the control problem for the case when the reference model is positive real. In this case the identification vector $\bar{\eta}(t)$ and the control parameter vector $\bar{\theta}(t)$, can be shown to be the same i.e.

$$\bar{\theta}(t) = \bar{\eta}(t). \tag{17}$$

For direct control the adaptive laws are

$$\dot{\bar{\theta}}(t) = e_1(t)\bar{w}_c(t)$$

For both direct and indirect control the plant together with the controller can be described by the n^{th} order vector differential equation:

$$\begin{aligned} \dot{x}_p &= A_p x_p + b_p[r(t) - \bar{\theta}(t)^T \bar{w}_c(t)] \\ \text{(Plant)} \qquad &= A_p x_p + b_p[r(t) - \bar{\theta}(t)^T \bar{w}_o(t)] \\ y_p(t) &= h^T x_p(t). \end{aligned} \tag{18}$$

The observer used in indirect control can be described by the differential equation

$$\dot{\hat{x}} = A_M\hat{x} + b_M[u(t) + \bar{\eta}^T(t)\bar{w}_o(t)] = A_M\hat{x}(t) + b_M r(t)$$

$$\text{(Observer)} \qquad y_o(t) = h^T\hat{x}(t) \tag{19}$$

where $y_o(t)$ is the estimate of $y_p(t)$ and $\hat{x}(t)$ is some linear transformation of the estimate of $x_p(t)$. Equation (19) is found to be a convenient minimal representation of the model. However, for estimating the state errors, the following non-minimal representation of both plant and model using a (3n-2) dimensional state space is found to be more suitable. The nonminimal reference model is obtained using the fact that the parameter vector $\bar{\theta}^*$ makes the plant transfer function identical to that of the model (by Lemma 1).

Defining $x_c^T \triangleq [x_p^T, v^{(1)^T}, v^{(2)^T}]$

and $\hat{x}_c^T \triangleq [\hat{x}_p^T, \hat{v}^{(1)^T}, \hat{v}^{(2)^T}]$

$$\dot{x}_c = A_c x_c + b_c[r - \phi_{\bar{\theta}}^T \bar{w}_c] \tag{20}$$

$$\dot{\hat{x}}_c = A_c\hat{x}_c + b_c[u(t) + \bar{\eta}^T(t)\bar{w}_o(t)] = A_c\hat{x}_c + b_c r(t)$$

where $\phi_{\bar{\theta}} = \bar{\theta} - \bar{\theta}^*$, $b^T = [0,0,\ldots 0,1]$ is an (n-1) dimensional vector, and the matrix A_c and vector b_c are given by

$$A_c = \left[\begin{array}{c|c|c} A_p + d_0^* b_p h^T & b_p c^{*T} & b_p d^{*T} \\ \hline b d_0^* h^T & \Lambda + b c^{*T} & b d^{*T} \\ \hline b h^T & 0 & \Lambda \end{array}\right] ; \quad b_c = \left[\begin{array}{c} b_p \\ \hline b \\ \hline 0 \end{array}\right]$$

and A_c is asymptotically stable.

The state error equation may now be described by the vector differential equation

$$\dot{\varepsilon}_c = \dot{x}_c - \dot{\hat{x}}_c = A_c\varepsilon_c + b_c[-\phi_{\bar{\theta}}^T \bar{w}_c] = A_c\varepsilon_c + b_c[-\phi_{\bar{\eta}}^T \bar{w}_o(t)] \tag{21}$$

and the adaptive laws for indirect control are

$$\dot{\bar{\eta}}(t) = \varepsilon_1(t)\bar{w}_o(t).$$

Since $\dot{\bar{\theta}}(t) = \dot{\bar{\eta}}(t)$ by equation (17) and $\bar{w}_c(t) = \bar{w}_o(t)$ by definition, it follows that in this case, for both direct and indirect control, either the control error $e_1(t)$ or the identification error $\varepsilon_1(t)$ can be used to adjust the parameters.

The simplicity of the adaptive laws is seen to be due to equation (17) by which the control and identification parameters are *identical* $(c_i(t) = \gamma_i(t);$ $d_j(t) = \delta_j(t);\ i = 1,2,\ldots n-1;\ j = 0,1,2,\ldots n-1)$.

If the initial values for the identification and control parameter vectors are chosen as

$$\bar{\eta}(0) = \bar{\theta}(0) = 0$$

the input u(t) to the plant may be represented as

$$u(t) = r(t) - \bar{\eta}^T(t)\bar{w}_c(t)$$

and the input to the reference model in the adaptive observer is r(t). This, in turn, implies that the output of the adaptive observer is exactly the same as the output of the reference model in the direct control case.

The above discussion is merely to emphasize that, when the reference model is positive real, there exists a parametrization of the plant such that the control parameters obtained using direct control also simultaneously identify the plant. The equivalence of the two approaches is obvious from Figure 6.

Comment: The observer output $\hat{y}_p(t)$ was seen to be the same as the output of the reference model in the case of direct control. This is due to the fact that the plant is being identified as a part of a closed loop and the same signal $\bar{\eta}^T(t)\bar{w}_o(t)$ is used both for identification and control.

Reference Model with Non-Positive Real Transfer Function:

It was shown in an earlier section that when the reference model has a transfer function $W_M(s)$ which is not positive real, the direct control problem can still be resolved by replacing all control parameters $\theta_i(t)$ by $P_L(\theta_i(t))$, where $P_L(\theta) = L(s)\theta(t)L^{-1}(s)$ and $W_M(s)L(s)$ is positive real. We shall use a similar procedure in this section for the indirect control problem.

When the reference model is not positive real, the observer structure suggested in the previous section, while still valid, is no longer useful directly for identifying the parameters $\bar{\eta}(t)$. However, if $\bar{\eta}(t)$ is replaced by $P_L(\bar{\eta}(t))$ where $W_M(s)L(s)$ is positive real, the same identification procedure can still be used. Figure 7 shows how the observer can be realized without using differentiators.

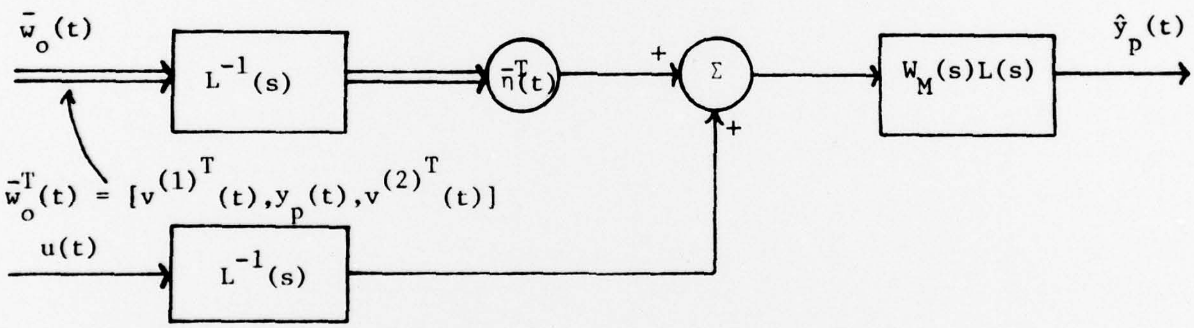

Figure 7

The controller still has the same structure as before with $\bar{\theta}(t) = \bar{\eta}(t)$. The estimate of the plant output as given by the observer is now

$$\hat{y}_p(t) = W_M(s)\{r(t) + (P_L(\bar{\eta}(t)) - \bar{\eta}(t))^T\bar{w}_o(t)\} \tag{22}$$

where the second term in the right-hand side has to tend to zero asymptotically if the adaptive control is stable. But $\hat{y}_p(t)$ as given by equation (22) is precisely the output of the reference model in the direct control case. More specifically, we can write the state equations for indirect control corresponding to

equations (20) in the previous case, as follows:

$$\begin{aligned}\dot{x}_c &= A_c x_c + b_c[L(s)\{L^{-1}(s)r(t) - L^{-1}(s)\phi_{\bar{\theta}}^T\bar{w}_o(t)\}] = \\ &= A_c x_c + b_{PR}[L^{-1}(s)r(t) - L^{-1}(s)\phi_{\bar{\theta}}^T\bar{w}_o(t)]\end{aligned} \tag{23}$$

where $h^T(sI-A_c)^{-1}b_c = W_M(s)$ and $h^T(sI-A_c)b_{PR} = W_M(s)L(s)$ is positive real.

$$\begin{aligned}\dot{\hat{x}}_c &= A_c\hat{x}_c + b_{PR}[L^{-1}(s)r(t) + L^{-1}\{P_L(\phi_{\bar{\theta}}) - \phi_{\bar{\theta}}\}^T\bar{w}_o(t)] = \\ &= A_c\hat{x}_c + b_{PR}[L^{-1}(s)r(t) + \{\phi_{\bar{\theta}}^T L^{-1} - L^{-1}\phi_{\bar{\theta}}^T\}\bar{w}_o(t)]\end{aligned} \tag{24}$$

From equations (23) and (24) the identification error vector $\varepsilon_c(t)$ is governed by the differential equation

$$\begin{aligned}\dot{\varepsilon}_c = A_c\varepsilon_c + b_{PR}\phi_{\bar{\theta}}^T L^{-1}\bar{w}_o(t) &= A_c\varepsilon_c + b_{PR}\phi_{\bar{\theta}}^T\bar{\zeta}_o(t) \\ \text{where } \bar{\zeta}_o(t) &\triangleq L^{-1}(s)\bar{w}_o(t).\end{aligned} \tag{25}$$

Equation (25) above implies that the specific parametrization of the plant used to design the observer yields an observer output which is indistinguishable from the output of the reference model in the direct control case. By emphasizing identification prior to control, we have merely restructured the entire configuration of the overall adaptive system. In this form, the adaptive loop, though equivalent, is considerably simpler to implement than in the direct control case.

Comment: It has been shown that the error equations for both direct and indirect control are the same. Consequently, the adaptive laws for updating the parameters are also identical.

In direct control [6], two transfer functions of degree (n-m-1) were needed for the adjustment of every parameter in the controller in addition to the two auxiliary signal generators, when the model is not positive real. The scheme proposed here uses only one such transfer function for the adjustment of each parameter and a

total of 2n transfer functions as shown in Figure 8. A further reduction in the number of such filters is also possible as shown in Figure 9, where four auxiliary signal generators and three transfer functions $L^{-1}(s)$ are used.

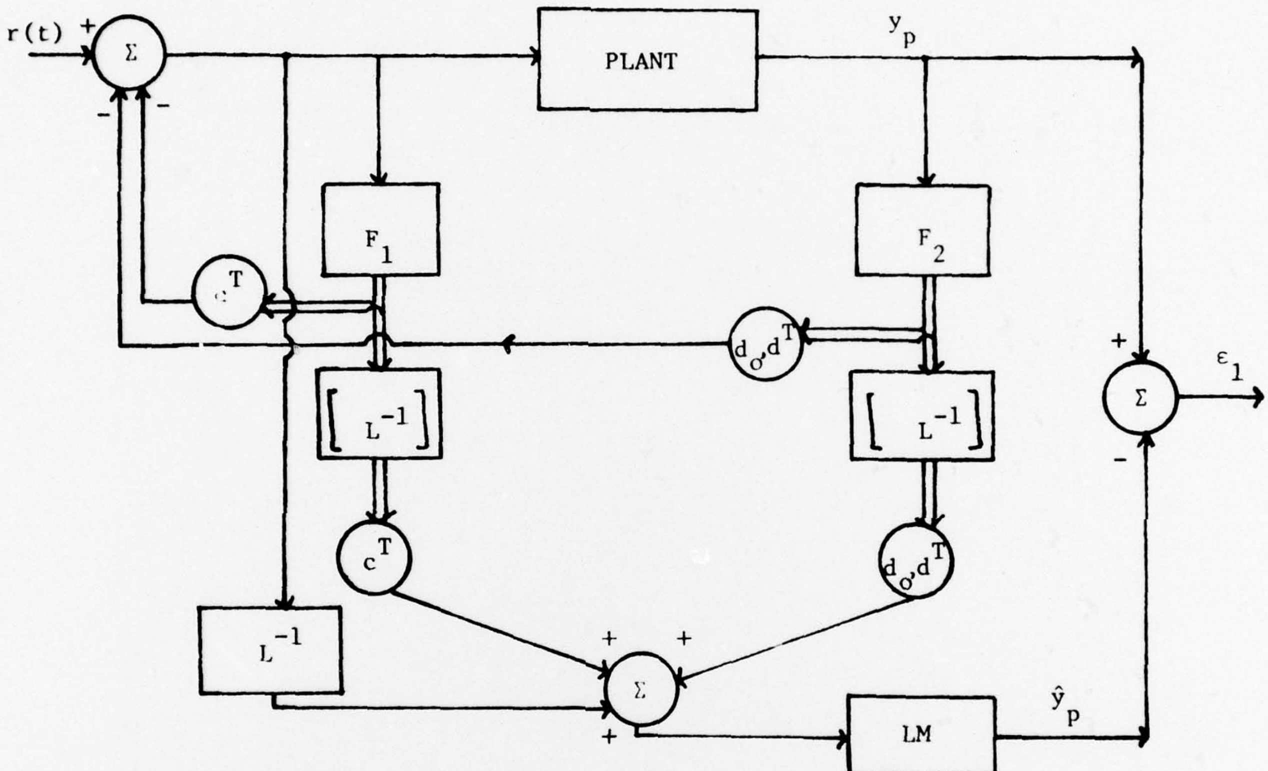

Figure 8

Simulation Results: The new scheme proposed in the previous section, while mathematically identical to that proposed in [6] is much simpler to implement. Simulation results on the digital computer clearly demonstrate that the method can be applied in practical situations for the control of unknown unstable plants.

Six typical examples are presented in this section and are shown in Figures 10 to 12. The plant has a fourth order transfer function and is unstable in three of the six cases simulated. Model transfer functions with one, two and three zeros and four poles are considered. Identification and control parameters have zero initial values and a square wave reference input is used in all cases.

In direct control the primary aim is to drive the output error to zero

Plant: $\dfrac{s^3 + 8s^2 + 20s + 16}{s^4 + 7s^3 + 11s^2 + 5s + 3}$

Model: $\dfrac{s^3 + 6s^2 + 11s + 6}{s^4 + 8.75s^3 + 25.75s^2 + 29.8125s + 11.25}$

Input: Square Wave

Amplitude: 2

Frequency: 10

Note: y_M in Figures 10-12 is the output of the reference model with input r(t).

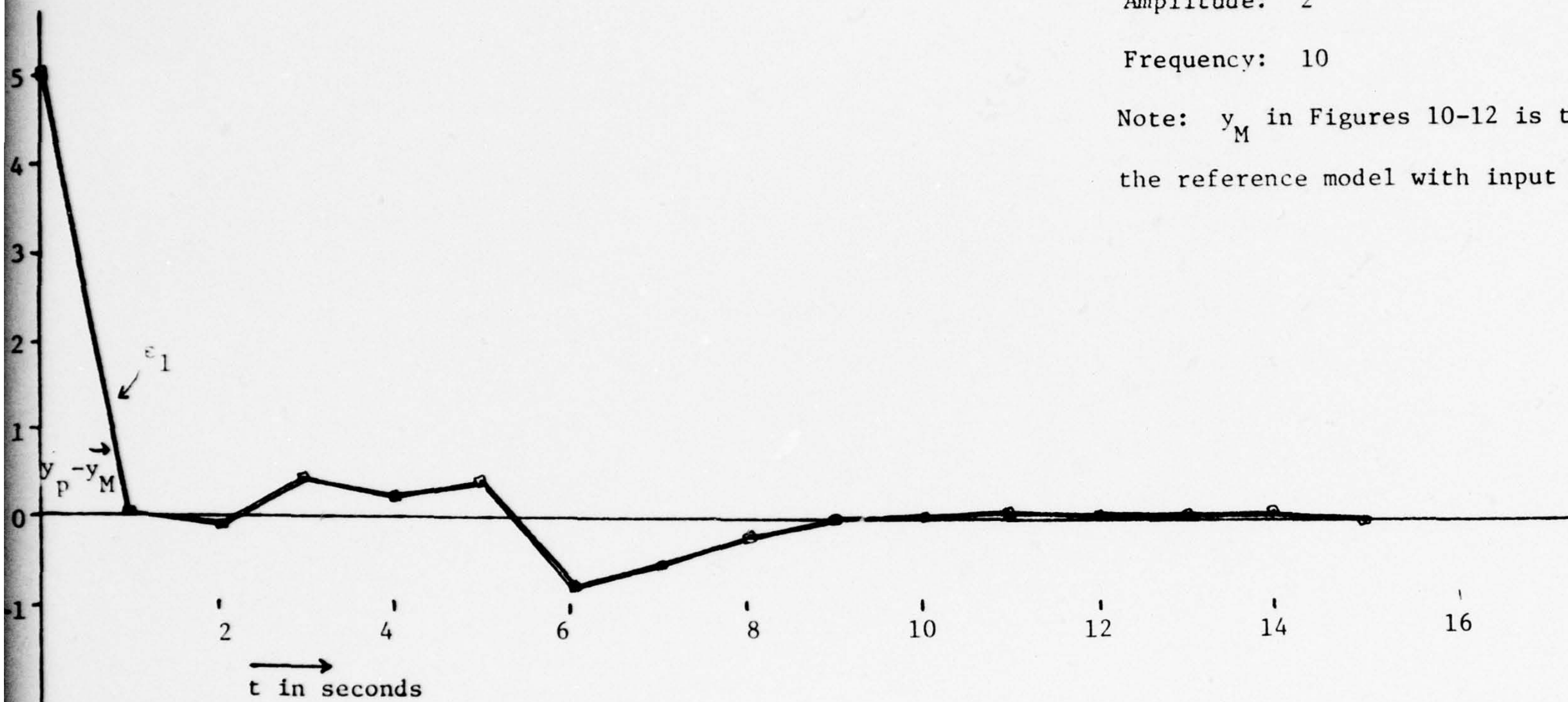

Figure 10a

Plant: $\dfrac{s^3 + 8s^2 + 20s + 16}{s^4 + 7s^3 + 11s^2 - 5s - 3}$

Model: $\dfrac{s^3 + 6s^2 + 11s + 6}{s^4 + 8.75s^3 + 25.75s^2 + 29.8125s + 11.25}$

Input: Square Wave

Amplitude: 2

Frequency: 10

INDIRECT CONTROL OF UNSTABLE PLANT

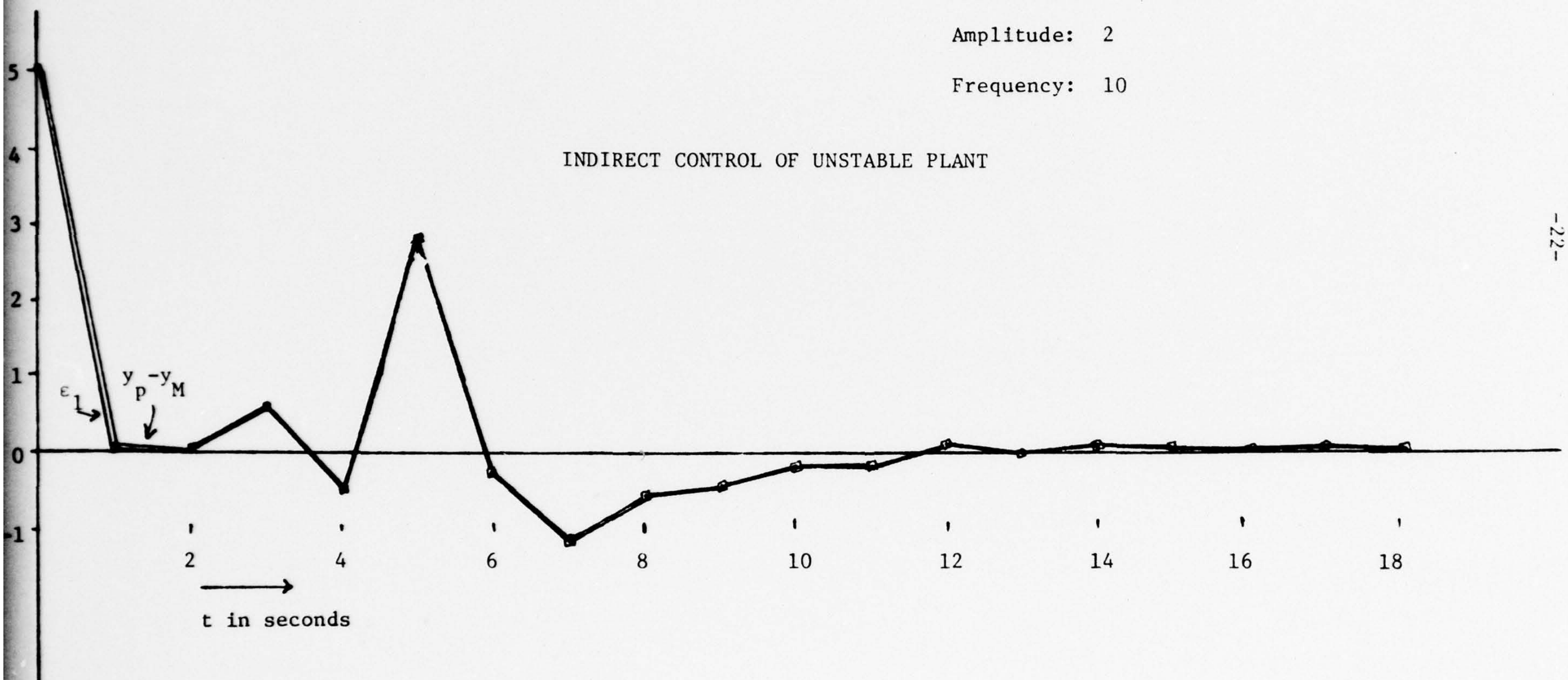

Figure 10b

Plant: $\dfrac{s^3 + 8s^2 + 20s + 16}{s^4 + 7s^3 + 11s^2 + 5s + 3}$

Model: $\dfrac{s^2 + 4s + 3}{s^4 + 8.75s^3 + 25.75s^2 + 29.8125s + 11.25}$

Input: Square Wave

Frequency: 10

Amplitude: 2

$L(s) = s + 2$

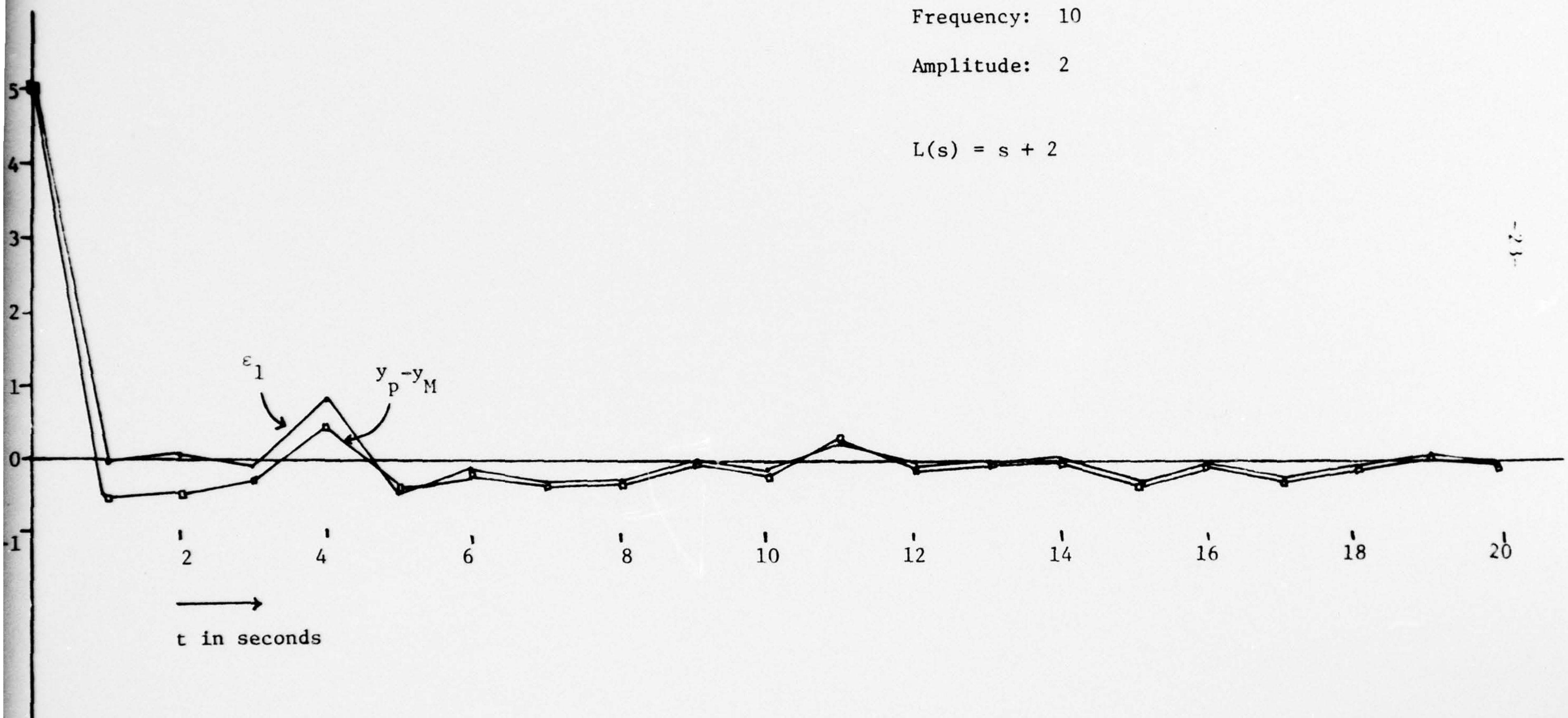

Figure 11a

Plant: $\dfrac{s^3 + 8s^2 + 20s + 16}{s^4 + 7s^3 - 11s^2 - 5s - 3}$

Model: $\dfrac{s^2 + 4s + 3}{s^4 + 8.75s^3 + 25.75s^2 + 29.8125s + 11.25}$

Input: Square Wave

Amplitude: 2

Frequency: 10

$L(s) = s + 2$

INDIRECT CONTROL OF UNSTABLE PLANT

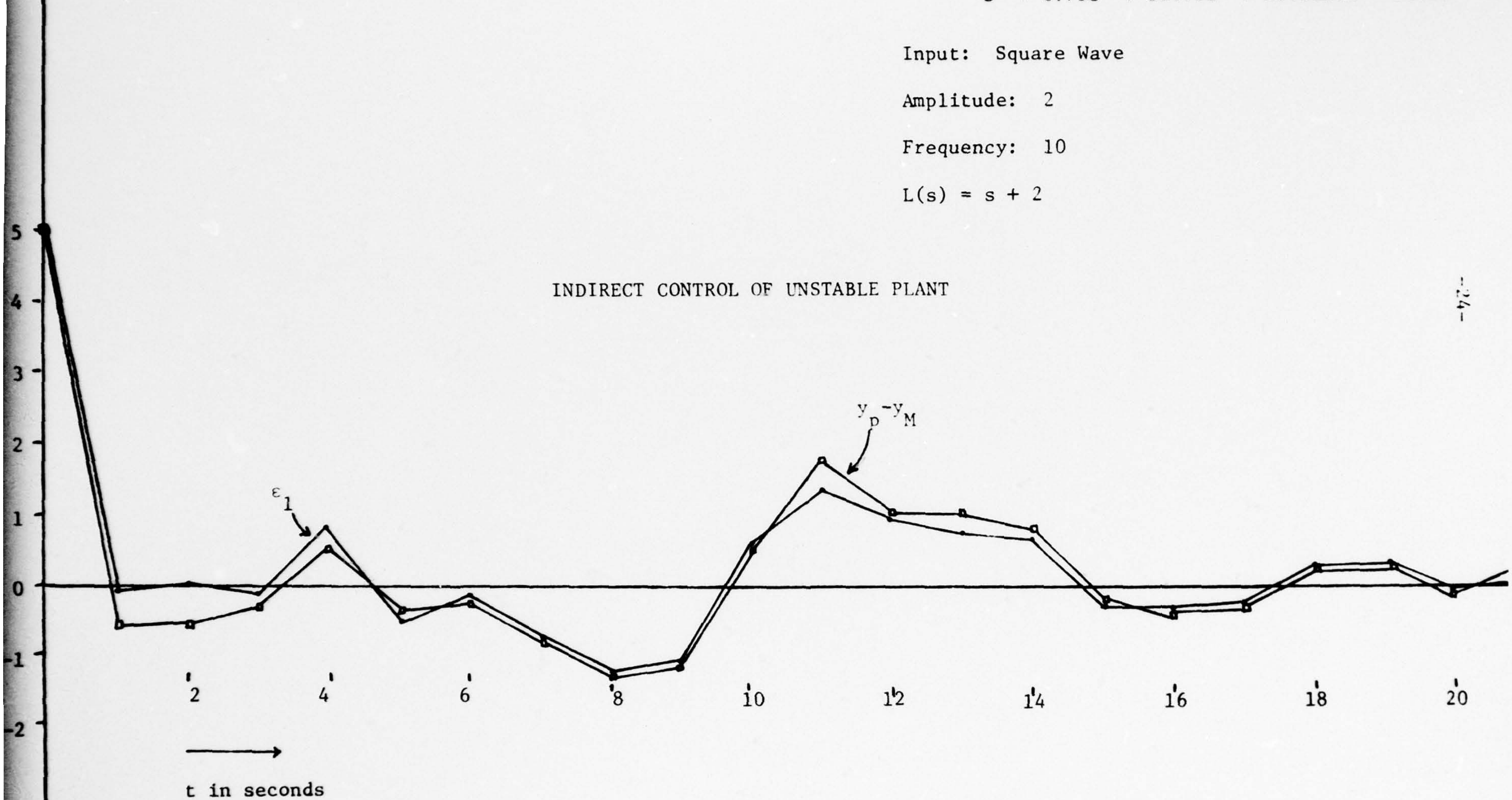

Figure 11b

Plant: $\dfrac{s^3 + 8s^2 + 20s + 16}{s^4 + 7s^3 + 11s^2 + 5s + 3}$

Model: $\dfrac{(s+3)}{s^4 + 8.75s^3 + 25.75s^2 + 29.8125s + 11.25}$

Input: Square Wave

Frequency: 10

Amplitude: 1

$L(s) = s^3 + 6s^2 + 11s + 6$

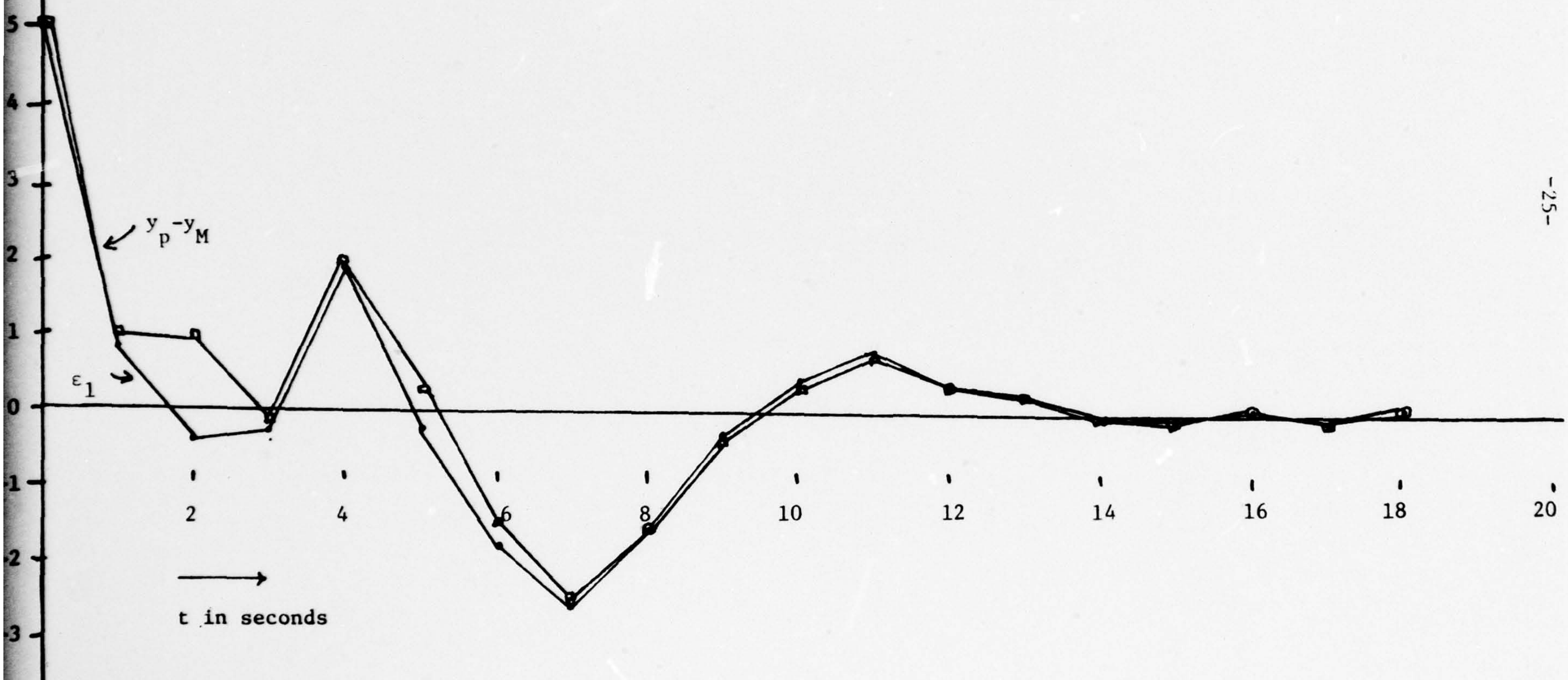

Figure 12a

Plant: $\dfrac{s^3 + 8s^2 + 20s + 16}{s^4 + 7s^3 + 11s^2 + 5s - 3}$

Model: $\dfrac{(s+3)}{s^4 + 8.75s^3 + 25.75s^2 + 29.8125s + 11.25}$

Input: Square Wave

Frequency: 10

Amplitude: 1

$L(s) = s^3 + 6s^2 + 11s + 6$

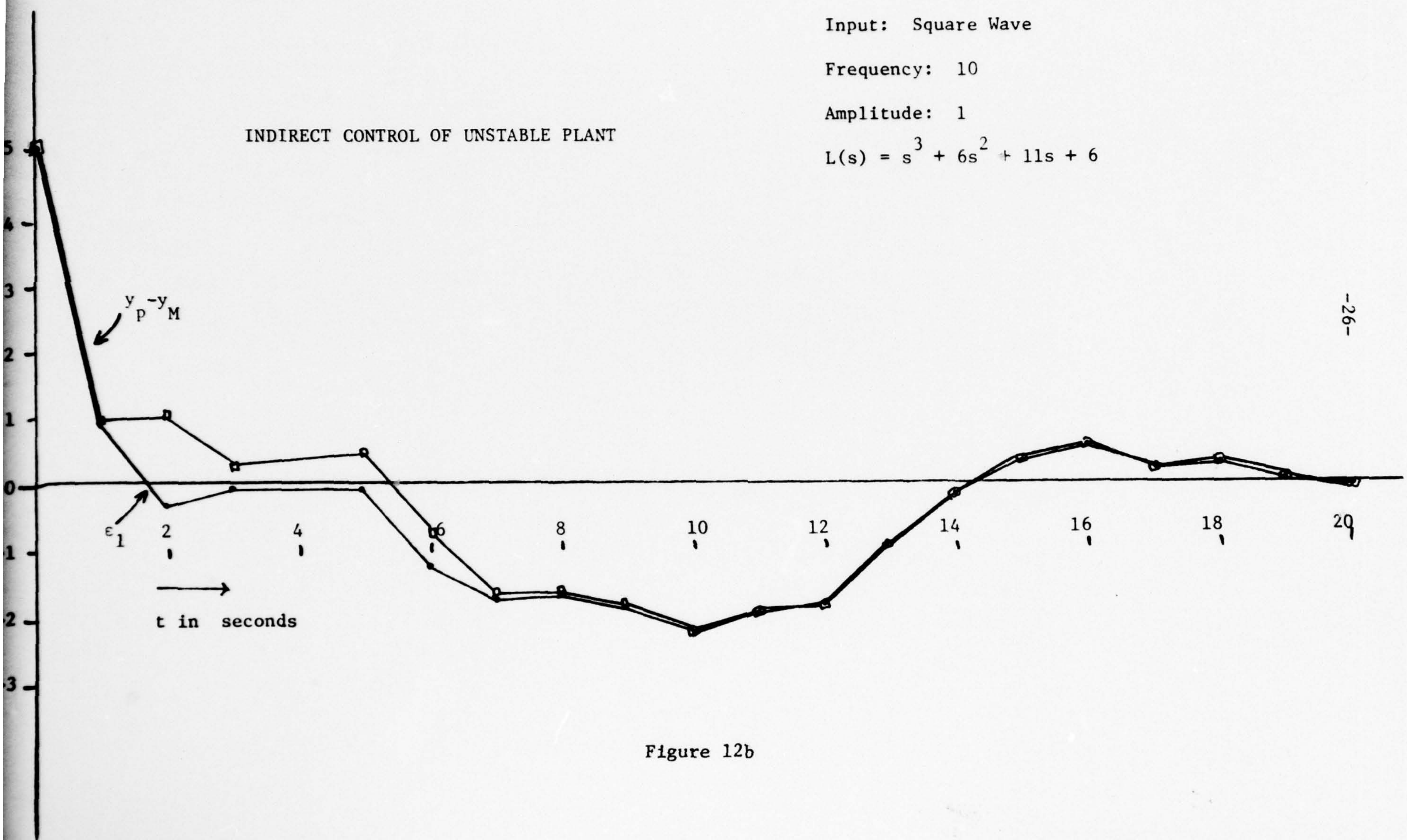

Figure 12b

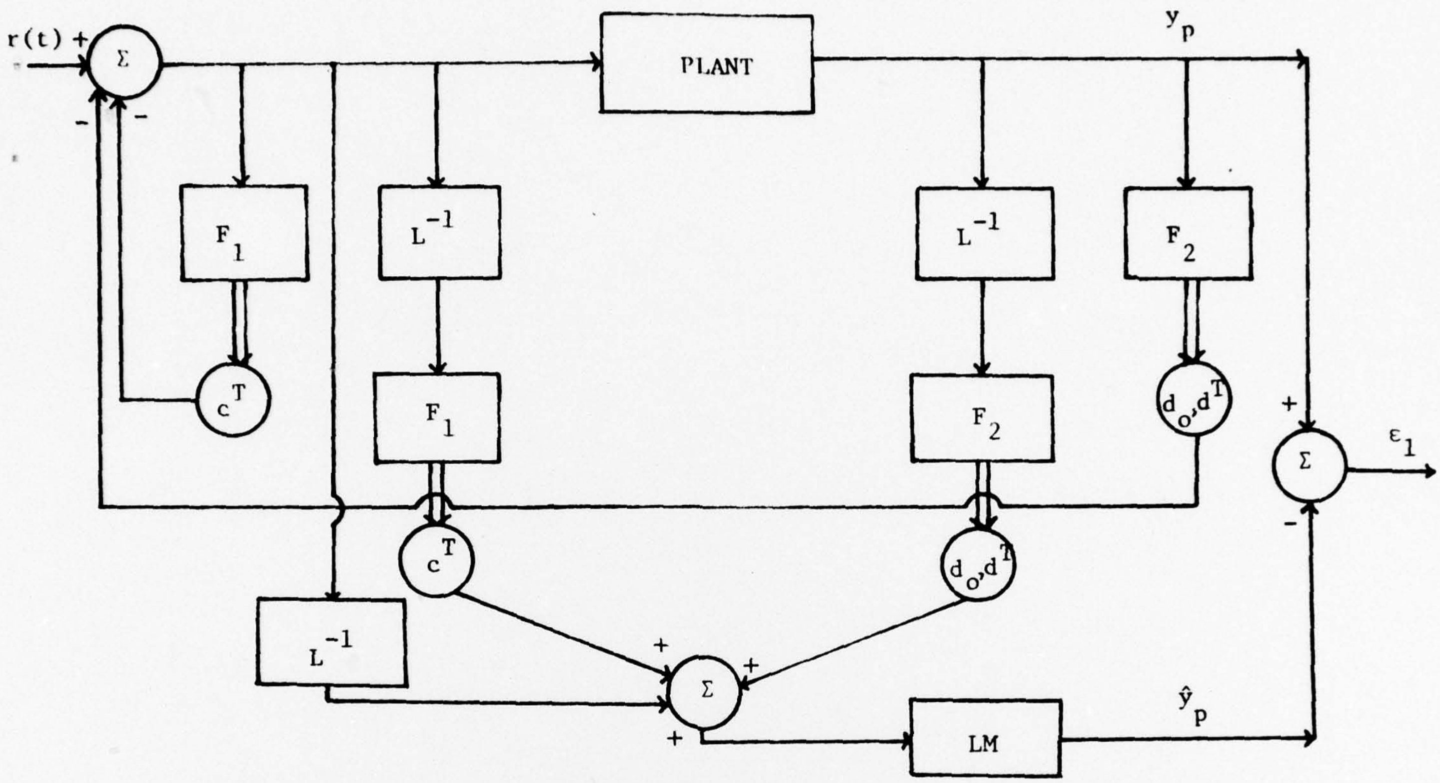

Figure 9

whether or not the control parameters converge to their desired values. In indirect control, it has always been assumed that the convergence of the identification parameters to their true values is important for successful control. However, for the particular plant parametrization used in this paper, we have shown that the error equations for both cases are identical and here it is not surprising that even in indirect control, as the simulations show, the identification and control errors tend to zero asymptotically without perfect identification.

Stability Problem of Indirect Control:

The convergence of observer parameters has been proven in the past only for bounded inputs (a reasonable assumption in the case of observers). However, in the control problem there is no guarantee that the feedback loop will be stable and hence that the input u(t) to the plant will be bounded. It is this fact that raises the principal question regarding the convergence of both observer and controller parameters.

The boundedness of the identification error $\varepsilon_1(t)$ is assured whether or not the adaptive loop is stable. However, the possibility exists that both plant and observer outputs are unbounded even while their difference is bounded. Our main concern is, therefore, to demonstrate the stability of the feedback loop.

In [6] a conjecture was made that when the control error is bounded that the plant and model outputs would also remain bounded. A similar conjecture can also be made in the indirect control case i.e. if the identification error is bounded, the plant and observer outputs must also be bounded. Figure 12, which shows the plant output $y_p(t)$ and its relation to the identification error $\varepsilon_1(t)$ is found to be identical to Figure 5a in [6] relating control error to $y_p(t)$.

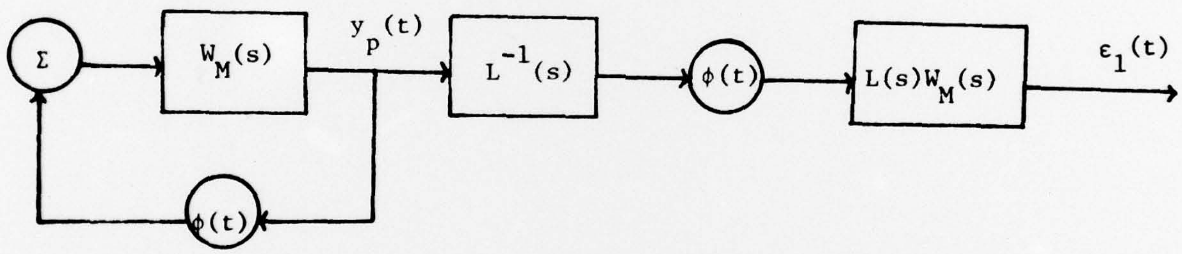

Figure 13

The above comments imply that if the conjecture made by Narendra and Valavani (1977) is correct, the stability problems of both direct and indirect control can be resolved simultaneously and stable adaptive control can be achieved using either the control error $e_1(t)$ or the identification error $\varepsilon_1(t)$.

Acknowledgement

The work reported here was supported by the Office of Naval Research under contract N00014-~~67~~ 76-C-0017

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